

REŠENJA

1) $(-6\sqrt{\frac{1}{4}} + \frac{\sqrt{36} \cdot \sqrt{0,16}}{2}) : \sqrt{25} = (-6 \cdot \frac{1}{2} + \frac{6 \cdot 0,4}{2}) : 5 = (-3 + 1,2) : 5 = (-1,8) : 5 = -0,36$

2) $5\sqrt{5} - \frac{25}{\sqrt{5}} - 3\sqrt{5} = 2\sqrt{5} - \frac{25}{\sqrt{5}} = 2\sqrt{5} - \frac{25\sqrt{5}}{5} = 2\sqrt{5} - 5\sqrt{5} = -3\sqrt{5}$

3) $(\sqrt{18} - 0,2\sqrt{75} + 2\sqrt{45}) \cdot \sqrt{5} = (\sqrt{18} \cdot \sqrt{5} - 0,2\sqrt{75} \cdot \sqrt{5} + 2\sqrt{45} \cdot \sqrt{5}) = (\sqrt{90} - 0,2\sqrt{375} + 2\sqrt{225}) = (\sqrt{90} - 0,2 \cdot 5\sqrt{3} + 2 \cdot 15) = (\sqrt{90} - \sqrt{3} + 30)$

4) $\sqrt{121} - \sqrt{\frac{16}{25}} \cdot (1\frac{2}{3}) - \sqrt{1 + \frac{9}{16}} = 11 - \frac{4}{5} \cdot \frac{5}{3} - \sqrt{1 + \frac{9}{16}} = 11 - \frac{4}{3} - \sqrt{\frac{25}{16}} = 11 - \frac{4}{3} - \frac{5}{4} = 11 - \frac{16}{12} - \frac{15}{12} = 11 - \frac{31}{12} = 11 - 2,58 = 8,42$

5) $(5x-3y)(5x+3y) - (5x-3y)^2 - x \cdot y^2 = (5x)^2 - (3y)^2 - (25x^2 - 30xy + 9y^2) - x \cdot y^2 = 25x^2 - 9y^2 - 25x^2 + 30xy - 9y^2 - x \cdot y^2 = 30xy - 18y^2 - x \cdot y^2 = 30 \cdot \frac{3}{2} - 18 \cdot (\frac{3}{2})^2 - \frac{3}{2} \cdot (\frac{3}{2})^2 = 45 - 40,5 - 1,125 = 3,375$

6) $P(x) = x^4 + x^3 - 3x^2 - x + 1$
 $Q(x) = x^3 - 3x^2 - x + 2$
 $R(x) = P(x) - Q(x) = x^4 + x^3 - 3x^2 - x + 1 - (x^3 - 3x^2 - x + 2) = x^4 + x^3 - 3x^2 - x + 1 - x^3 + 3x^2 + x - 2 = x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x-1)(x+1)(x^2 + 1)$

7) $\frac{a^2 + 5ab}{a + 5b} = \frac{a^2(a + 5b)}{a + 5b} = a^2, a + 5b \neq 0$

8) $\frac{4x^2 + 4xy + y^2}{2x + y} = \frac{(2x + y)^2}{2x + y} = 2x + y, 2x + y \neq 0$

9) $\frac{x^3 - y^3}{x^2 + xy + y^2} = \frac{(x-y)(x^2 + xy + y^2)}{x^2 + xy + y^2} = x - y$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

10) $\frac{99^2 - 1}{99 \cdot 100} = \frac{(99-1)(99+1)}{(99-45)(99+45)} = \frac{98 \cdot 100}{10 \cdot 144} = \frac{98}{144} = 0,68$

11) $A = 9 + 3^2(-5+4)^2 + (-1)^{100} = 9 + 9 \cdot (-1)^2 + 1 = 9 + 9 \cdot 1 + 1 = 19$

$B = 4 \cdot 2^3 - 3 \cdot 2^3 + (-1)^{51} = 4 \cdot 8 - 3 \cdot 8 - 1 = 32 - 24 - 1 = 7$

$A - B = 19 - 7 = 12$

12) $(5+x)(u+x) = (2u-x)(3-x) + 1$
 $5u + 5x + ux + x^2 = 6u - 2x - 3x + x^2 + 1$
 $5u + 5x + ux + x^2 = 6u - 5x + x^2 + 1$
 $5u - 6u + 5x + 5x + ux - x^2 + x^2 = 1$
 $-u + 10x + ux = 1$
 $-u + 10x + ux - 10x - 10x = 1 - 20x$
 $-u - 10x = 1 - 20x$
 $-u = 1 - 20x + 10x = 1 - 10x$
 $u = 10x - 1$

8) $\frac{x+2y+4}{4} = 3 \quad | \cdot 4$
 $x+2y+4 = 12$
 $x+2y = 8$
 $5x - \frac{y}{2} = 18 \quad | \cdot 2$
 $10x - y = 36$
 $x+2y = 8$
 $11x = 44 \Rightarrow x = \frac{44}{11} = 4$
 $4+2y = 8$
 $2y = 4$
 $y = 2$
 $(x,y) = (4,2)$

9) $\frac{(3x+5y-16)}{2} - \frac{(4x-3y-2)}{5} = 4 \quad | \cdot 10$
 $(5x+7y-14) - (2x-3y-2) = 40$
 $3x+10y-12 = 40$
 $3x+10y = 52$
 $9x+30y = 156$
 $9x+10y = 52$
 $20y = 104$
 $y = 5,2$
 $3x+10 \cdot 5,2 = 52$
 $3x+52 = 52$
 $3x = 0$
 $x = 0$
 $(x,y) = (0,5,2)$

10) $(x+1)^2 + (y-2)^2 = x^2 + y^2$
 $(x^2 + 2x + 1) + (y^2 - 4y + 4) = x^2 + y^2$
 $2x + 5 = 4y$
 $2x - 4y = -5$
 $4x - 8y = -10$
 $-4x - 2y = -5$
 $-10y = -15 \Rightarrow y = \frac{15}{10} = 1,5$
 $2x - 4 \cdot 1,5 = -5$
 $2x - 6 = -5$
 $2x = 1$
 $x = 0,5$
 $(x,y) = (0,5, 1,5)$

11) $(x^2)^2 \cdot (x^2)^2 = x^4 \cdot x^4 = x^8$
 $x^2 \cdot (x^2)^2 = x^2 \cdot x^4 = x^6$
 $\frac{x^8}{x^6} = x^2$
 1) $\frac{81^{m+1}}{3^m \cdot 3^{2m-6}} = \frac{(3^4)^{m+1}}{(3^1)^m \cdot (3^2)^{2m-6}} = \frac{3^{4(m+1)}}{3^m \cdot 3^{4m-12}} = \frac{3^{4m+4}}{3^{5m-12}} = 3^{4m+4-5m+12} = 3^{-m+16} = 3^{16-m}$
 2) $\frac{16^4 \cdot 4^{16}}{8^2} = \frac{(2^4)^4 \cdot (2^2)^{16}}{(2^3)^2} = \frac{2^{16} \cdot 2^{32}}{2^6} = \frac{2^{48}}{2^6} = 2^{42}$
 3) $\frac{0,25^6 \cdot 4^6}{0,5^4} = \frac{(0,25)^6 \cdot 4^6}{0,5^4} = (\frac{1}{4})^6 \cdot (2^2)^6 \cdot (\frac{1}{2})^{-4} = (\frac{1}{4})^6 \cdot 2^{12} \cdot 2^4 = (\frac{1}{4})^6 \cdot 2^{16} = (\frac{1}{2})^{12} \cdot 2^{16} = 2^4 = 16$

14. X - время работы
 I $30\% \cdot X = \frac{30}{100} X = 0,3X$
 II $\frac{2}{3} \cdot (X - 0,3X)$
 III 120 часов
 I - II + III = X
 $0,3X + \frac{2}{3}(X - 0,3X) + 120 = X$
 $\frac{3}{10}X + \frac{2}{3}(X - \frac{3}{10}X) + 120 = X$
 $\frac{3}{10}X + \frac{2}{3}X - \frac{2}{10}X + 120 = X$
 $21X + 20X - 6X + 8400 = 70X$
 $8400 = 70X - 25X$
 $8400 = 45X$
 $X = 240$ часов

15. $X:Y = 5:4$ $Y:Z = 8:15$
 $Z:5 = 6:2$ $Y:9 = 8:15$ $X:Y = 5:4$
 $2Z = 30$ $15Y = 98$ $X:Y = 5:4$
 $2Z = 18$ $15Y = 98$ $4X = 5 \cdot \frac{24}{5}$
 $Z = 9$ $Y = \frac{24}{5}$ $4X = 24$
 $X = 6$

17. $4800 = X$
 $4800 \text{ грн} \rightarrow 100\%$
 $X \text{ грн} \rightarrow 70\%$
 $4800 : 100 = X : 70$
 $100 \cdot X = 4800 \cdot 70$
 $X = \frac{4800 \cdot 70}{100}$
 $X = 3360 \text{ грн}$

18. $X = 5600 \cdot 100$
 $X = 560000 \text{ грн}$
 $560000 - 540000 = 20000 \text{ грн}$

16. $O = 52 \text{ км}$
 $a: b: c = 2:5:6$
 $a = 2k$
 $b = 5k$
 $c = 6k$
 $O = a + b + c$
 $52 = 2k + 5k + 6k$
 $52 = 13k$
 $k = 4$
 $a = 8 \text{ км}$
 $b = 20 \text{ км}$
 $c = 24 \text{ км}$

19. $2200 \text{ грн} \rightarrow 100\%$
 $2200 \cdot 90\% = 1980 \text{ грн}$
 $2200 \cdot 95\% = 2090 \text{ грн}$
 $1980 - 0,95 \cdot 2090 = X$
 $1980 - 1980,5 = X$
 $X = -0,5 \text{ грн}$

20. $X = 75\% \cdot 110 = 82,5$
 $X = \frac{75}{100} \cdot 110 = 82,5$
 $X = \frac{82,5}{100} \cdot 100 = 82,5$
 $X = \frac{82,5 \cdot 100}{100} = 82,5$
 $X = 80$

21. 700 км/ч
 $700 - 70 = 630 \text{ км/ч}$
 $630 \cdot X = 9 \cdot 700$
 $X = \frac{9 \cdot 700}{630} = 10 \text{ ч}$

22. $X = 2100 \text{ грн}$
 $2625 \text{ грн} \rightarrow 125\%$
 $X = 2100$

$2625 \rightarrow 100\%$
 $2100 \rightarrow X\%$
 $2625 : 100 = 2100 : X$
 $2625 \cdot X = 2100 \cdot 100$
 $X = \frac{210000}{2625}$
 $X = 80\%$

23. $X:15 = 6:8$
 $X:8 \cdot 9 = 15 \cdot 8 \cdot 6$
 $X = \frac{15 \cdot 8 \cdot 6}{8 \cdot 9}$
 $X = \frac{15 \cdot 2}{3} = 10 \text{ грн}$

26. $h = \frac{a\sqrt{3}}{2}$ $P = \frac{a^2\sqrt{3}}{4}$
 $P_0 = \frac{a_0^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3}$
 $P = \frac{144\sqrt{3}}{4} = 36\sqrt{3} \text{ см}^2$

24. $y = kx + n$
 $A(-1; 3)$
 $2x - y + 13 = 0$
 $-y = -2x - 13$
 $y = 2x + 13$
 $k = 2$
 $y = 2x + n$
 $3 = 2(-1) + n$
 $3 = -2 + n$
 $n = 5$
 $y = 2x + 5$

25. a) $d = a\sqrt{2}$
 $d = 4\sqrt{2} \text{ см}$
 $a\sqrt{2} = 4\sqrt{2}$
 $a = 4 \text{ см}$
 $O = 4 \cdot a = 16 \text{ см}$
 $P = a^2 = 16 \text{ см}^2$
 $P = 16 \text{ см}^2$

$O = 3 \cdot a$
 $O = 3 \cdot 12 = 36 \text{ см}$
 $P_0 = \frac{a_0^2\sqrt{3}}{4} = \frac{12^2\sqrt{3}}{4} = 36\sqrt{3} \text{ см}^2$
 $P = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3} \text{ см}^2$
 $P_u = \frac{a_u^2\sqrt{3}}{4} = \frac{2^2\sqrt{3}}{4} = \sqrt{3} \text{ см}^2$

27.

$b=3\text{cm}$
 $c=5\text{cm}$
 $h=4\text{cm}$
 $P = \frac{a \cdot b}{2} \cdot h$
 $P = \frac{3 \cdot 4}{2} \cdot 4 = 6 \text{ cm}^2$
 $a = b + x$
 $a = 3 + 2$
 $a = 5 \text{ cm}$

28.

$d_1 = 16 \text{ cm}$
 $P = 96 \text{ cm}^2$
 $O = ?$
 $h = ?$
 $P = a \cdot h$
 $96 = 16 \cdot h$
 $h = 6 \text{ cm}$
 $P = \frac{a \cdot b}{2}$
 $96 = \frac{a \cdot b}{2}$
 $a \cdot b = 192$
 $a^2 = b^2 + c^2$
 $a^2 = 64 + 36$
 $a^2 = 100$
 $a = 10 \text{ cm}$

29.

$P_0 = a^2$
 $P_1 = \pi r^2$
 $P_2 = 4 \cdot \frac{a^2}{4} = a^2$
 $P_3 = 4 \cdot \frac{a^2}{4} = a^2$
 $P_4 = 4 \cdot \frac{a^2}{4} = a^2$
 $P_5 = \pi r^2$
 $P_6 = a^2$
 $P_7 = 4 \cdot \frac{a^2}{4} = a^2$
 $P_8 = \pi r^2$
 $P_9 = a^2$
 $P_{10} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{11} = \pi r^2$
 $P_{12} = a^2$
 $P_{13} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{14} = \pi r^2$
 $P_{15} = a^2$
 $P_{16} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{17} = \pi r^2$
 $P_{18} = a^2$
 $P_{19} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{20} = \pi r^2$
 $P_{21} = a^2$
 $P_{22} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{23} = \pi r^2$
 $P_{24} = a^2$
 $P_{25} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{26} = \pi r^2$
 $P_{27} = a^2$
 $P_{28} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{29} = \pi r^2$
 $P_{30} = a^2$
 $P_{31} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{32} = \pi r^2$
 $P_{33} = a^2$
 $P_{34} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{35} = \pi r^2$
 $P_{36} = a^2$
 $P_{37} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{38} = \pi r^2$
 $P_{39} = a^2$
 $P_{40} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{41} = \pi r^2$
 $P_{42} = a^2$
 $P_{43} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{44} = \pi r^2$
 $P_{45} = a^2$
 $P_{46} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{47} = \pi r^2$
 $P_{48} = a^2$
 $P_{49} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{50} = \pi r^2$
 $P_{51} = a^2$
 $P_{52} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{53} = \pi r^2$
 $P_{54} = a^2$
 $P_{55} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{56} = \pi r^2$
 $P_{57} = a^2$
 $P_{58} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{59} = \pi r^2$
 $P_{60} = a^2$
 $P_{61} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{62} = \pi r^2$
 $P_{63} = a^2$
 $P_{64} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{65} = \pi r^2$
 $P_{66} = a^2$
 $P_{67} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{68} = \pi r^2$
 $P_{69} = a^2$
 $P_{70} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{71} = \pi r^2$
 $P_{72} = a^2$
 $P_{73} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{74} = \pi r^2$
 $P_{75} = a^2$
 $P_{76} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{77} = \pi r^2$
 $P_{78} = a^2$
 $P_{79} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{80} = \pi r^2$
 $P_{81} = a^2$
 $P_{82} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{83} = \pi r^2$
 $P_{84} = a^2$
 $P_{85} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{86} = \pi r^2$
 $P_{87} = a^2$
 $P_{88} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{89} = \pi r^2$
 $P_{90} = a^2$
 $P_{91} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{92} = \pi r^2$
 $P_{93} = a^2$
 $P_{94} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{95} = \pi r^2$
 $P_{96} = a^2$
 $P_{97} = 4 \cdot \frac{a^2}{4} = a^2$
 $P_{98} = \pi r^2$
 $P_{99} = a^2$
 $P_{100} = 4 \cdot \frac{a^2}{4} = a^2$

30. a)

$\frac{(x-5)(2-x)}{(x-1)} > 0$
 $x-5=0 \rightarrow x=5$
 $2-x=0 \rightarrow x=2$
 $x-1=0 \rightarrow x=1$

	$-\infty$	1	2	5	$+\infty$
$x-5$	-	-	-	0	+
$2-x$	+	+	0	-	-
$x-1$	-	0	+	+	+
$\frac{(x-5)(2-x)}{(x-1)}$	+	-	+	-	+

$x \in (-\infty, 1) \cup (2, 5)$

b)

$x^2 + 2x - 15 < 0$
 $a=1, b=2, c=-15$
 $x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x_{1/2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1}$
 $x_{1/2} = \frac{-2 \pm \sqrt{4 + 60}}{2}$
 $x_{1/2} = \frac{-2 \pm \sqrt{64}}{2}$
 $x_{1/2} = \frac{-2 \pm 8}{2}$
 $x_1 = \frac{-2 + 8}{2} = 3$
 $x_2 = \frac{-2 - 8}{2} = -5$

$x^2 + 2x - 15 < 0$ (не равенство)
 $x \in (-5, 3)$